

Unit 15

Day 5

IN 2012, I DID NOT GET THROUGH THIS ENTIRE LESSON... MAY NEED TO ADDRESS THIS BY GIVING THE NOTES ONE DAY AND THEN DOING EXAMPLES THE NEXT.

Rational Root theorem:

If p/q is a rational number written in lowest terms, and if p/q is a zero of f , a polynomial function with integer coefficients, then p is a factor of the constant term and q is a factor of the leading coefficient.

Corollary of the rational root theorem:

If the leading coefficient of a polynomial function with integral coefficients is 1, then any rational zeros of the function are integers and factors of the constant terms.

Factor completely and identify the zeros.

1) $f(x) = 4x^3 + 16x^2 + 19x + 6$

of possible zeros:

factors of p: $\pm 1, \pm 2, \pm 3, \pm 6$

factors of q: $\pm 1, \pm 2, \pm 4$

possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{1}{4}$
 $\pm \frac{3}{2}, \pm \frac{3}{4}$

| | 4 | 16 | 19 | 6 | |
|----------------|---|----|----|----|------------|
| 0 | | | | 6 | |
| 1 | 4 | 20 | 39 | 45 | UB |
| -1 | 4 | 12 | 7 | -1 | btw 0 & -1 |
| $-\frac{1}{2}$ | 4 | 14 | 12 | 0 | ✓ |

$$f(x) = (x + \frac{1}{2})(4x^2 + 14x + 12)$$

$$f(x) = (2x + 1)(2x^2 + 7x + 6)$$

$$f(x) = (2x + 1)(2x + 3)(x + 2)$$

zeros: $-\frac{1}{2}, -\frac{3}{2}, -2$

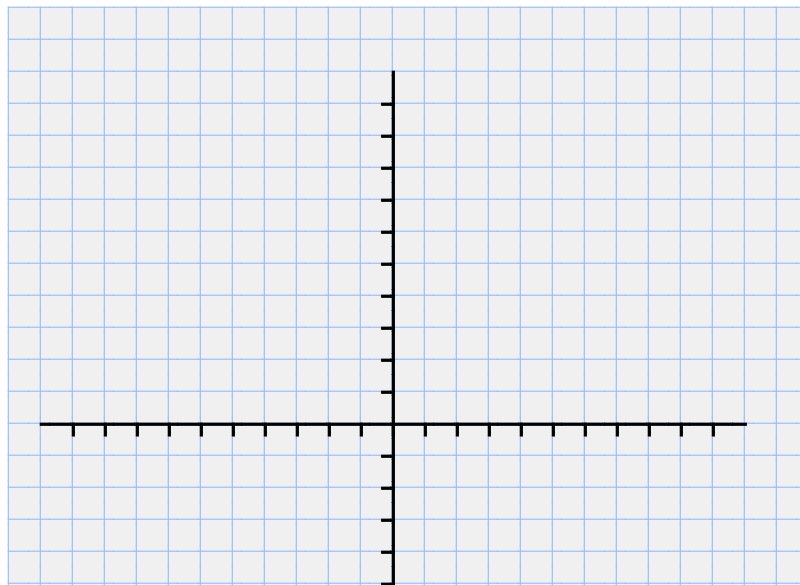
Boundedness Theorem:

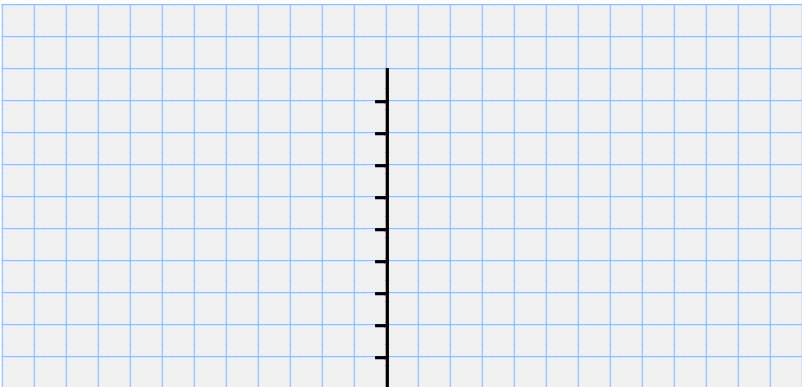
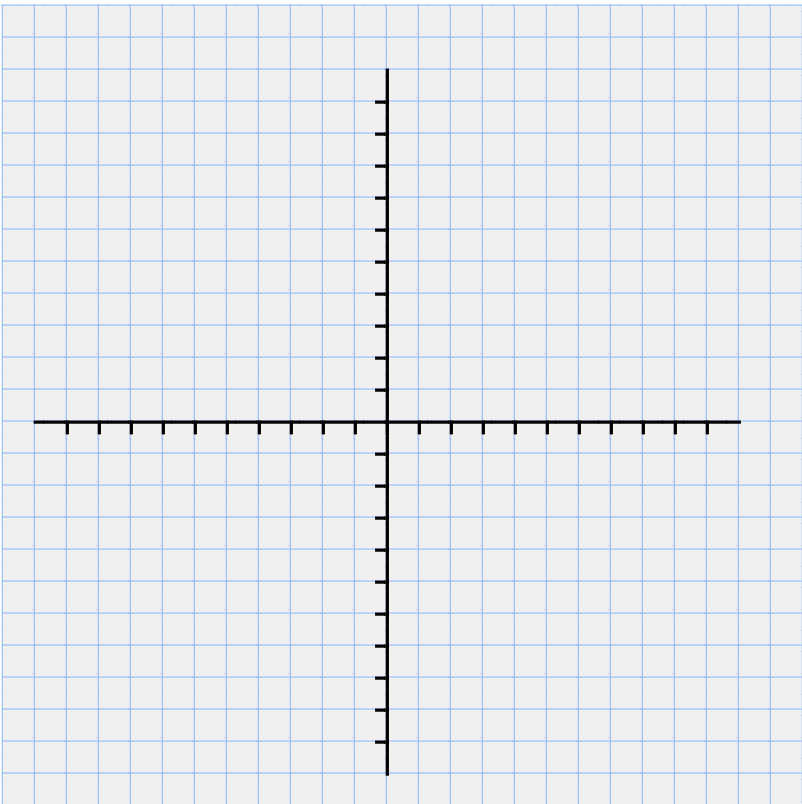
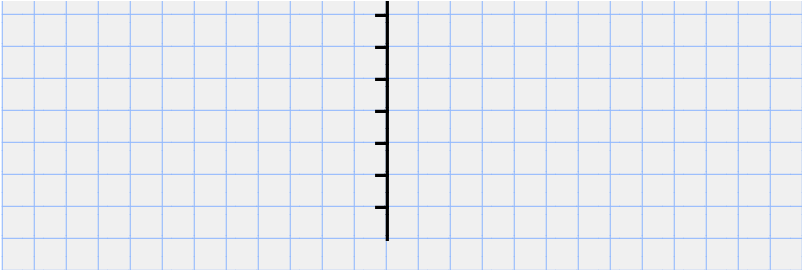
Let $f(x)$ be a polynomial of degree $n \geq 1$ with real coefficients and with a positive leading coefficient. If $f(x)$ is divided synthetically by :

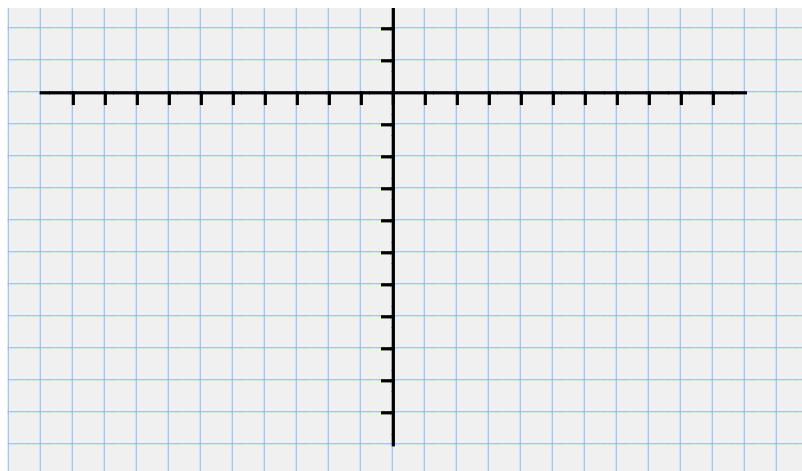
- a) $k > 0$ and all numbers in the result are non-negative, then $f(x)$ has no zero greater than k .
- b) $k < 0$ and the numbers in the result alternate signs (0 pos. or neg as needed, then $f(x)$ has no zero less than k .

Intermediate value theorem:

If $f(x)$ is polynomial with only real coefficients , and if for real #'s a and b , the values of $f(a)$ and $f(b)$ are opposite in sign then there exists at least one zero between a and b .







2) $f(x) = 2x^4 + 15x^3 + 28x^2 - 9x - 36$

of possible zeros: 4

factors of p: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 18, \pm 36, \pm 12$

factors of q: $\pm 1, \pm 2$

possible rational zeros: $\pm 1, \pm \frac{1}{2}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm 4, \pm 6, \pm 9, \pm \frac{9}{2}, \pm 18, \pm 36, \pm 12$

| | | | | | | |
|---|---|----|----|----|-----|----|
| | 2 | 15 | 28 | -9 | -36 | |
| 0 | | | | | -36 | |
| 1 | 2 | 17 | 45 | 36 | 0 | UB |

$$f(x) = (x-1)(2x^3 + 17x^2 + 45x + 36)$$

| | | | | | |
|-------------------|---|----|----|-----|------------|
| | 2 | 17 | 45 | 36 | |
| 0 | | | | 36 | |
| -1 | 2 | 19 | 64 | 100 | |
| -1 | 2 | 15 | 30 | 6 | |
| -2 | 2 | 13 | 19 | -2 | > INT. VAL |
| 3 2 | 2 | 14 | 24 | 0 | ✓ |

$$f(x) = (x + \frac{3}{2})(x-1)(2x^2 + 14x + 24)$$

$$f(x) = (2x+3)(x-1)(x^2 + 7x + 12)$$

$$f(x) = (2x+3)(x-1)(x+4)(x+3)$$

zeros $-\frac{3}{2}, 1, -4, -3$
 HW pg 300 25-28 all & pg 312-313 33-38, 41-44 all